ON THE DYNAMICS OF OPEN CLUSTERS[1]

It has been pointed out in the literature that, due to several causes, open star clusters dissipate with time. For instance, Rosseland showed that when external stars move through a cluster, they cause perturbation of the motion of the stars in the cluster and could transfer enough momentum to individual stars to cause their escape from the cluster's gravitational field. In this way the cluster will lose stars gradually, i.e., it will dissipate. According to Rosseland the time needed for the star cluster to dissipate following the outlined mechanism is 10^{10} years. However, as pointed out by the present author in the supplement to the Russian edition of Rosseland's book, there is another factor that makes the life of an open cluster even shorter: the stars in the cluster may have close encounters with each other. As a result, they exchange kinetic energy and gradually tend toward the most probable distribution, i.e., a Maxwell-Boltzmann distribution. And this, as we shall see shortly, also causes the dissipation of the cluster.

The relaxation time, i.e., the time in which the encounters of the stars in a cluster will lead to statistical equilibrium, is given approximately by the formula:

$$\tau = \frac{3\sqrt{2}}{32\pi n} \cdot \frac{v^3}{G^2 m^2 \log\left(\frac{\rho}{\rho_0}\right)}, \tag{1}$$

where n is the number of stars per unit volume, m the stellar mass, G the gravitational constant, v the average stellar velocity in the cluster, ρ the radius of the cluster, and ρ_0 the distance at which the potential energy of two stars is equal to the average kinetic energy of stars in the cluster, i.e.,

$$\rho_0 = \frac{2Gm}{v^2}. \tag{2}$$

[1]English translation originally published by Kluwer Academic Publishers in *Dynamics of Star Clusters*, Jeremy Goodman and Piet Hut (eds.), 1985, pp. 521-524, IAU © 1985. Used here with permission of the International Astronomical Union.

Formula (1) was derived for the case of stars with equal masses.

The average velocity v enters formula (1) both explicitly and through ρ_0. To find v we assume that for shorter time intervals the cluster is stationary. We can do this since the time necessary to change the distribution law for the stars in the cluster, considered as a system in phase space, is large compared with the time necessary for a star to cross the cluster. In the case of a stationary system consisting of particles interacting according to Newton's law, using the virial theorem we can write

$$U = 2T, \tag{3}$$

where U is the absolute value of the potential energy of the system, and T is its kinetic energy.

The exact formula for U is:

$$U = \frac{1}{2} \sum_{i \neq k} \frac{G m^2}{r_{ik}},$$

where we assume again that all stellar masses are equal, and r_{ik} denotes the distance between the i-th and the k-th star. We replace all of the r_{ik}'s with their mean harmonic value which is apparently close to the radius of the cluster ρ. Then approximately

$$U = \frac{1}{2} \frac{GN(N-1)\,m^2}{\rho},$$

where N is the total number of stars in the cluster. For $N \gg 1$,

$$U = \frac{1}{2} \frac{GN^2\,m^2}{\rho}.$$

On the other hand

$$2T = Nmv^2.$$

Therefore, the virial theorem assumes the form:

$$v^2 = \frac{GNm}{2\rho}. \tag{4}$$

Comparing (4) with (2), we find that

$$\log\left(\frac{\rho}{\rho_0}\right) = \log\left(\frac{N}{4}\right). \tag{5}$$

Substituting (4) and (5) in (1) and taking into account that

$$n = \frac{N}{\frac{4}{3}\pi\rho^3},$$

we find

$$\tau = \frac{2}{16\log\left(\frac{N}{4}\right)} \cdot \sqrt{\frac{N\rho^3}{Gm}}. \tag{6}$$

Assuming that for a typical cluster $N = 400$, $\rho = 2$ parsecs, $m = 2 \times 10^{33}$ g, we find the relaxation time to be $\tau \approx 4 \times 10^7$ years.

A result of the evolution of the distribution towards a Maxwell-Boltzmann distribution is the presence of stars with kinetic energy larger than the escape energy for the cluster. Such stars tend to leave the cluster. The whole question is, what is the percentage of such stars in a cluster with the Boltzman distribution? If this percentage is small, then the dissipation of the cluster as a result of this process will be very slow. It is apparent that the ratio of the number of stars which can escape in a relaxation time τ to the total number of stars in the cluster is equal to

$$P = \frac{\int_{\varepsilon_0}^{\infty} \exp\left(-\frac{\varepsilon}{\theta}\right)\sqrt{\varepsilon}\,d\varepsilon}{\int_0^{\infty} \exp\left(-\frac{\varepsilon}{\theta}\right)\sqrt{\varepsilon}\,d\varepsilon}, \tag{7}$$

where ε_0 is the escape energy, and θ is equal to two thirds of the average kinetic energy, i.e.,

$$\theta = \frac{2}{3}\frac{T}{N} = \frac{1}{3}\frac{U}{N}. \tag{8}$$

On the other hand, the mean value of ε_0, i.e., the escape energy, is equal to

$$\varepsilon_0 = \overline{\sum_k \frac{Gm^2}{r_{ik}}} = \frac{2U}{N}, \tag{9}$$

where the line over the sum indicates an average over i. Comparing (8) with (9) we find:

$$\varepsilon_0 = 6\theta.$$

Substituting in (7), we obtain the approximation:

$$P \approx \frac{\exp\left(-\frac{\varepsilon_0}{\theta}\right)\theta\sqrt{\varepsilon_0}}{\frac{1}{2}\sqrt{\pi}\,\theta^{3/2}} = 2\,e^{-6}\sqrt{\frac{6}{\pi}},$$

i.e., one hundredth of the total number of stars should escape a cluster in a relaxation time. Therefore, the dissipation time of the cluster should be of the order of several billion years.

This result was obtained for a cluster consisting of stars with equal masses. Therefore, these numbers are applicable only to stars of the cluster with masses close to the average mass of the stars in the cluster. For stars with masses two to three times less than the average, the escape time will be of the order of a few hundred million years. Remarkably, the open clusters are known to possess rather few dwarfs.

Let us assume for a moment that the open clusters we observe are different stages in the evolution of one and the same cluster. Since the stars escaping from the cluster carry away positive kinetic energy, the total cluster energy

$$H = T - U \tag{10}$$

should decrease with the transition from richer to poorer clusters.

If we substitute (3) in (10), we find

$$H = \frac{1}{2}U. \tag{11}$$

Therefore, under the above assumption, U should increase. The data in the article by Orlova show that no increase of U with the decrease of N is observed.

Another possible hypothesis is that all clusters were formed approximately at the same epoch (perhaps even at the epoch of the formation of the galaxy itself). Then the evolution of rich clusters with large diameters should be slower. Among other things, these rich and large clusters should contain a higher percentage of dwarfs. It seems to the author that this conclusion is supported by observation. That is, the clusters h and χ Persei are both rich and contain a high percentage of dwarfs. On the other hand, a number of poor clusters have hardly any dwarfs. Hence, it becomes clear that to make further conclusions it is of great interest to determine not only the luminosity function for different clusters, but also the total energy H. According to (11), H can be determined from the absolute value of the potential energy.

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